LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

SECOND SEMESTER – **APRIL 2013**

ST 2814/2811 - ESTIMATION THEORY

Date : 26/04/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Part – A

Answer all the questions: (10 x 2 = 20)

1) Define a statistic with an example.
2) Examine whether the family N $(0,\sigma^2)$ is complete.
3) Define CAN estimator.
4) Obtain the sufficient estimator when a random sample is from b(1, θ), 0<θ<1.
5) Explain the concept of likelihood function.
6) Obtain the method of moments estimator of θ when a random sample is from f(x)= θ $e^{-\theta x}$, x>0.
7) State Rao-Blackwell theorem.
8) Write the unbiased estimator of σ^2 when a random sample of size n is drawn from N(μ, σ^2).
9) Define ancillary statistic.
10) What is the best estimator of θ in Bayesian estimation with respect to
    i) Squared error loss function?
    ii) Absolute error loss function?

Part – B

Answer any 5 questions: (5 x 8 = 40)

11) Let X_1 , X_2 be independent random variables whose distribution depend on θ. Then show that $I_{(X_1,X_2)}(\theta) = I_{(X_1)}(\theta) + I_{(X_2)}(\theta)$

12) State and prove a necessary and sufficient condition for an estimator to be UMVUE using uncorrelatedness.

13) Let X_1 , X_2 ,… X_n be a random sample from f(x)= $\frac{1}{\sigma} e^{\frac{-(x-\mu)}{\sigma}}$, μ<x. Obtain the mle of μ and σ.

14) Let X_1 , X_2,… X_n be a random sample from N(μ, σ^2). Obtain the minimal sufficient statistic.

15) Let the r.v X have

$$P(x) = \begin{cases} \theta, & x = -1 \\ (1-\theta)^2\theta^x, & x = 0,1,2,\ldots \; 0 < \theta < 1 \end{cases}$$

Show that the family is not complete but boundedly complete.

16) Let X_1 , X_2,… X_n be a random sample from N(0, θ^2). Obtain the Cramer-Rao lower bound for estimating θ^2.

17) Let δ_0 be a fixed member of U_g. Then prove that $U_g = \{\delta_0 + u | u \in U_0\}$.

18) Let X_1 , X_2,… X_n be a random sample from N(μ, 1). Let μ have the prior distribution N(0, 1). Obtain the Bayes estimator of μ.

PART – C

Answer any 2 questions 2 x 20= 40

19)a) State and prove Lehman-Scheffe theorem.

b) Let X_1 , X_2,… X_n be a random sample from b(1, θ), 0<θ<1. Obtain the UMVUE of θ(1-θ)/n.

c) Let δ_1 and δ_2 be the UMVUEs of g_1(θ) and g_2(θ) respectively. Show that a_1 δ_1+ a_2 δ_2 is the UMVUE of a_1 g_1(θ)+ a_2 g_2(θ). (8+8+4)

20)a) State and prove Basu's theorem.

b) Let X_1 , X_2,… X_n be a random sample from N(μ, σ^2). Show that $\bar{X}$ and S^2 are independent.

c) Let X_1 , X_2,… X_n be a random sample from f(x)= $\frac{1}{\theta} e^{\frac{-x}{\theta}}$, x>0. Show that the complete sufficient statistic is independent of X_1 /$\sum X_i$.

(8+6+6)

21)a) Let X_1 , X_2,… X_n be a random sample from E(θ, 1). Show that MLE of θ is not CAN but consistent. Suggest a CAN estimator for θ.

b) Let X_1 , X_2,… X_n be a random sample from b(1, θ), 0<θ<1. Let θ have the prior Beta(α, β). Obtain the Bayes estimator of

b) θ

ii) θ(1-θ)

(10+10)

22) a) Explain EM algorithm in detail.

b) Explain Jacknife estimator in detail.

(10+10)

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